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COMMENT

Monte Carlo renormalisation of the five-dimensional Ising model

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Abstract. We analyse the five-dimensional Ising model using Monte Carlo methods.

To test Stauffer's scaling hypothesis for block spin magnetisation (the majority rule) for five dimensions we analysed the Ising model using Monte Carlo methods. Finite-size scaling and block spin renormalisation for Ising and similar models usually only work if 'hyperscaling' is correct (Brezin 1982), i.e. below four dimensions. However, Binder *et al* (1985) found the cumulant scaling of block spin renormalisation to work even for the five-dimensional Ising model (see also Binder 1985). We test here two other scaling hypotheses for five dimensions:

$$M_b = f(X) \qquad X = b^{1/\nu}(T - T_c)/T_c$$

for the equilibrium magnetisation of superspins arising from majority-rule renormalisation of cells of length b (Stauffer 1984), and

$$M_b = g(Y) \qquad Y = t/b^z$$

for the time-dependent superspin magnetisation at $T = T_c$ (Jan *et al* 1983, Kalle 1984).

By extrapolation of the effective critical temperatures T_c^{eff} of different large square lattices (T_c^{eff} against L^{-2}) the critical temperature of an infinite square lattice was evaluated as $J/k_BT_c = 0.114$, consistent with other studies (Guttmann 1981). To be sure that our program works correctly we also used it in three dimensions and compared our data with Stauffer's, with which they agreed.

Although we used a relatively small system ($L = 8$) our renormalisation (cell sizes $b = 1, 2, 4$) showed that the scaling hypothesis in the static case ($X = b^{1/\nu}(T - T_c)/T_c$) is hardly correct in five dimensions. In figures 1 and 2 we can see that the deviation between the magnetisation of the renormalised system ($b = 2, 4$) and unrenormalised system ($b = 1$) in our scaling plot is much greater than in three dimensions. This effect is for temperatures, which are low enough, independent of the size of the system (we compared $L = 4$ with $L = 8$).

We also tried to test the time-dependent scaling hypothesis of Jan *et al* and Kalle. In this dynamic case we were not able to smooth out all fluctuations, because of the small size of our lattice ($L = 8$). We could therefore neither prove nor disprove the dynamic scaling hypothesis.

To summarise, at least the static scaling hypothesis does not seem to work in five dimensions, for $M_b(T)$.

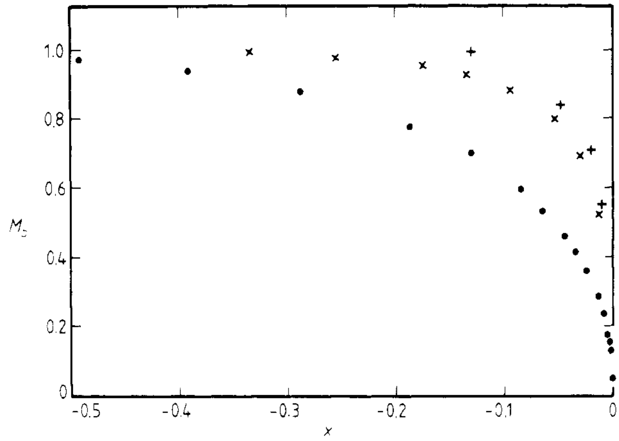

Figure 1. Renormalised magnetisations plotted against scaled temperature difference. If scaling were valid, data for different b would follow the same curve. $J/k_B T_c^{eff} = 0.116$. ●, $b = 1$; ×, $b = 2$, +, $b = 4$.

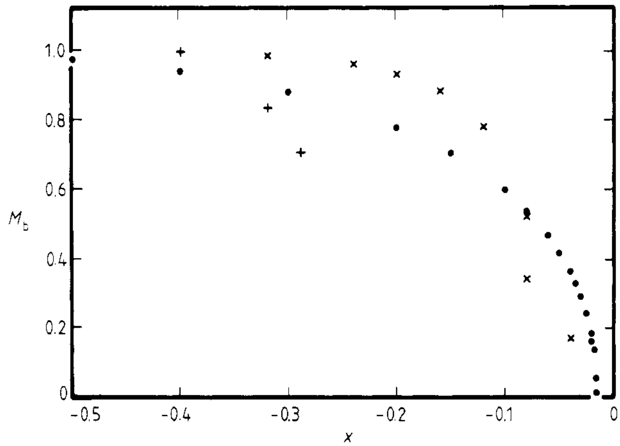

Figure 2. Same as in figure 1, with $J/k_B T_c = 0.114$.

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